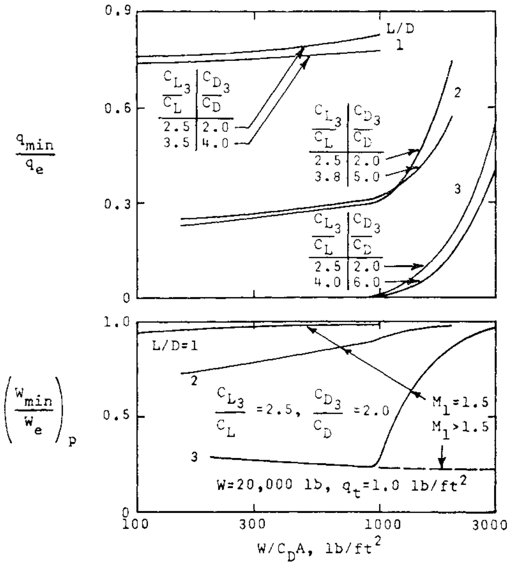

Fig. 3 Minimum dynamic pressures and weights vs ballistic coefficient.

representative of the pitch-up characteristics of lifting entry vehicles at subsonic speeds. $q_{\min}$ is nondimensionalized by an equilibrium dynamic pressure, q_e, for C_{L3} and C_{D3} conditions ($q_{\min}/q_e \approx 1$, if no zoom maneuver is used) defined by the equation

$$q_e = (W/C_DA)\ [(C_{D3}/C_D)^2 + (L/D)^2\ (C_{L3}/C_L)^2]^{-1/2}$$

It is seen that a zoom maneuver reduces $q_{\min}$ drastically for high-lifting vehicles ($L/D \geq 2$) that are not restricted by altitude limitations ($W/C_DA < 1000$ lb/ft²). $q_{\min} = 0$ is also obtainable for $L/D = 3$, $W/C_DA = 3000$ lb/ft², $C_{L3}/C_L = 2.5$, $C_{D3}/C_D = 2.0$, for instance, if M_1 is increased to 3.01 ($q_{\min} = 0$ infers that the vehicle can be pulled-up to $\gamma = 90°$ and glided to $M = q = 0$.).

To illustrate the magnitude of the effect of these reductions in q on terminal descent systems weights, parachute system weights were determined from the empirical relationships of Ref. 1 for an entry vehicle weight of 20,000 lb and a terminal q of 1 lb/ft². Figure 3 (bottom) shows $W_{\min}/W_e$, the ratio of the weight of a parachute system designed for deployment at $q_{\min}$ to that of one designed for deployment at q_e. Little advantage is obtained by use of the zoom maneuver for $L/D = 1$ class vehicles, but for vehicles with $L/D = 3$, a zoom maneuver can reduce the parachute system weight by about a factor near 4, yielding a weight saving of greater than 1000 lb for a 20,000-lb vehicle.

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Opening Distance of a Parachute

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Nomenclature

A_i, A_e = average effective inlet and exit areas, respectively, during the inflation process
A_{net} = $A_i - A_e$, net inlet area
D_0 = parachute nominal diameter
L = distance traveled during the inflation process
M_∞ = parachute Mach number at full inflation
S_p, S_{p_f} = parachute projected area and value at full inflation
t, t_f = time and time interval between snatch force and full inflation, respectively.
V_c = volume of parachute canopy
α = $V_c/A_{\text{net}}D_0$, dimensionless ratio
γ = ratio of specific heats
ρ_∞, ρ_T = freestream static and stagnation densities, respectively
ρ_{T_D} = stagnation density downstream of a normal shock
ρ_c = density inside canopy at full inflation

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Introduction

A KNOWLEDGE of parachute inflation characteristics is necessary to estimate parachute opening forces. French[1] suggested that the opening distance is a basic parameter in an analysis of parachute inflation characteristics. Based on theoretical considerations of the inflation of a parachute in incompressible flow and subsonic flight test data, he concluded that a given parachute should open in a fixed distance. Reference 2 indicates that the opening distance of a geometrically porous parachute is constant at all Mach numbers for infinite mass deployments; i.e., deployments during which the system velocity remains essentially constant. This Note contains an approximate compressible flow analysis and data from recent NASA programs[3–10] which indicate that although the opening distance may be relatively constant for incompressible flow, it increases with M_∞ for compressible flow. A technique is presented for predicting opening distances and inflation times for parachutes deployed supersonically based on their subsonic performance.

Approximate Analysis and Results

Berndt[11] indicates that a given parachute configuration will exhibit a characteristic canopy area growth with time, i.e., a characteristic curve for S_p/S_{p_f} vs t/t_f in incompressible flow. By assuming that this characteristic curve is independent of Mach number, one may say that during the inflation process some average value of effective inlet area, A_i, exists which is constant for a given parachute configuration. It is also assumed that a given parachute will have a constant value of effective exit area A_e, which will depend on the porosity and porosity distribution in the canopy. Therefore, there will be a net effective inlet area A_{net}, which is $A_i - A_e$. During the inflation process, a parachute "scoops out" a "column of air" of length L and average cross-sectional area A_i. Since part of this column of air A_eL exits the canopy during the inflation process, the mass of air filling the canopy is $\rho_\infty LA_{\text{net}}$. The mass of air in a fully inflated canopy is ρ_cV_c by definition of ρ_c and V_c. Since $\rho_\infty LA_{\text{net}} = \rho_cV_c$, the opening distance L can be expressed as

$$L = (\rho_c/\rho_\infty)V_c/A_{\text{net}} = (\rho_c/\rho_\infty)\alpha D_0 \qquad (1)$$

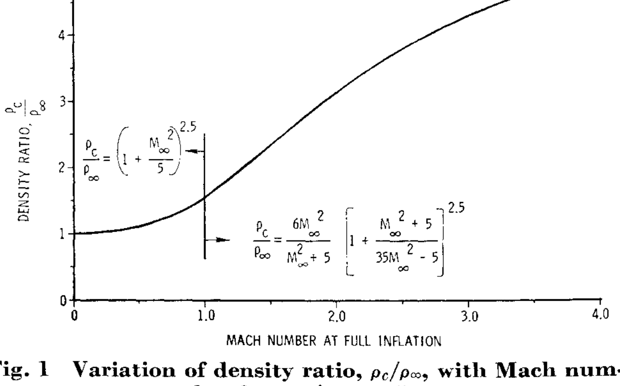

Fig. 1 Variation of density ratio, ρ_c/ρ_∞, with Mach number for air ($\gamma = 1.4$).

From this, it is seen that for a given parachute, L is proportional to the ratio of the density inside the canopy to the freestream density since V_c, A_{net}, and therefore α are constant.

The ratio ρ_c/ρ_∞ may be calculated for any Mach number if the following assumptions are made: 1) the velocity of the air inside the canopy is essentially zero and 2) for supersonic deployment, there is a normal shock wave in front of the canopy during inflation. Thus, for the subsonic case, the simple stagnation-to-static density ratio[12] for the freestream applies

$$\rho_c/\rho_\infty = \rho_T/\rho_\infty = \{1 + [(\gamma - 1)/2]M_\infty^2\}^{1/\gamma-1} \quad (2)$$

For supersonic velocities, the ratio of stagnation density behind a normal shock wave to freestream density applies. For a perfect gas,[12]

$$\frac{\rho_c}{\rho_\infty} = \frac{\rho_{TD}}{\rho_\infty} = \frac{(\gamma + 1)M_\infty^2}{(\gamma - 1)M_\infty^2 + 2}\left(1 + \frac{\gamma - 1}{2} M_D^2\right)^{1/\gamma-1} \quad (3)$$

where[12]

$$M_D^2 = [(\gamma - 1)M_\infty^2 + 2]/[2\gamma M_\infty^2 - (\gamma - 1)]$$

Using Eqs. (1-3), the opening distance may be determined in terms of αD_0 for any deployment conditions in any type of atmosphere if the values of γ and M_∞ are known. For the specific case of an infinite mass deployment, the opening time may be computed directly from the opening distance and deployment velocity. It should be noted that for infinite mass deployments at the same Mach number, the opening time will vary with atmospheric composition (value of γ) and the speed of sound. This may be important for considerations of performance in planetary atmospheres.

The value of α is most easily determined experimentally with several deployments (to get an average) at a low subsonic Mach number whereas Eq. (2) shows $\rho_c \simeq \rho_\infty$ so that $L \simeq \alpha D_0$. However, the value of α may be determined at any M_∞ by evaluating ρ_c/ρ_∞ from Eq. (2) or (3) and measuring L. Figure 1 shows ρ_c/ρ_∞ vs M_∞ for air ($\gamma = 1.4$), for which Eqs. (2) and (3) reduce to the forms shown in the figure.

Figure 2 shows opening distances determined from radar and camera data for several tests of disk-gap-band, modified ringsail, and cross parachutes[3–10] compared with infinite mass predictions from Ref. 2 and results from the present analysis. (Approximate deployment conditions for these tests are given in Table 1.) The disk-gap-band and modified ringsail parachutes had either 12.5 or 15% geometric porosity and generally had cloth permeability of about 150 scfm/ft² at 0.5 in. of water pressure differential. The cross parachutes had about the same cloth permeability as the other parachutes but the open area of the canopy in the flying shape is not known. However, the opening distance data for all three parachutes agree very well with the predicted values for $\alpha = 8$ and $\alpha = 10$ in the present analysis and are significantly greater than the maximum values predicted in Ref. 2 for supersonic velocities. For these data, the opening distance as a function of M_∞ may be represented by a line for $\alpha = 9$ with a scatter band of about ±25%. The data point for the disk-gap-band parachute deployed above Mach 3 indicates that the opening distance at high Mach numbers may be even greater than predicted. It is felt that this is a Mach number effect which is dependent upon the porosity. For a given value of porosity, there is a Mach number (termed the "critical Mach number" by Maynard[13]) above which there may be no shock wave in front of the canopy. For parachutes with 12–15% geometric porosity, the critical Mach number given in Ref. 13 is approximately 2.75. Although the approximate analysis assumes a shock wave in front of the canopy, it may still provide a reasonable estimate of the opening distance above the critical Mach number. More data are needed at the higher Mach numbers to define the limits of the analysis.

Table 1 Parachute deployment conditions

Parachute type	Nominal diameter, ft	System weight, lb	Approximate deployment Mach No.	Deployment altitude, ft
Cross	30	390	0.27	10,000
Cross	30	600	0.24	10,000
Cross	30	600	0.26	10,000
Cross	30	600	0.26	10,000
Cross	30	600	0.26	10,000
Cross	30	240	1.57	136,000
Cross	54.5	566	1.65	131,500
Modified ringsail	31.2	380	0.27	10,000
Modified ringsail	31.2	221	1.39	122,500
Modified ringsail	54.5	549	1.60	132,000
Disk-gap-band	40	394	0.27	10,000
Disk-gap-band	40	284	3.31	168,200
Disk-gap-band	40	280	1.91	140,000
Disk-gap-band	40	280	2.72	158,000
Disk-gap-band	64.7	557	1.59	133,500

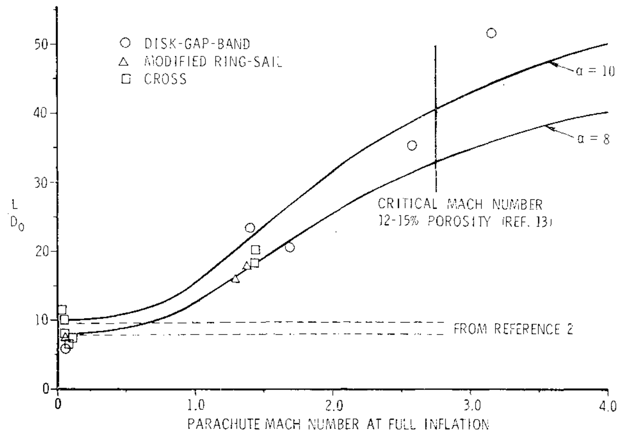

Fig. 2 Variation of opening distance with Mach number for disk-gap-band, modified ringsail, and cross parachutes.

Conclusion

The results presented indicate that the distance L travelled during the inflation of a parachute is a semiempirically predictable function of M_∞, the Mach number reached at full inflation. Measurements made at subsonic velocities can be used to predict L/D_0, within about ±25%, if the relation between M_∞ and deployment Mach number also is known. In the practical case, the predictability may be limited to the "critical" Mach number.

For the specific case of infinite mass deployment conditions, the inflation time may be calculated if the gas composition, deployment Mach number and velocity are known.

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Normalization and Convergence Acceleration for Indirect Optimization Methods

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THE mathematical descriptions of various optimal physical processes can be formulated as two-point boundary value problems.[1,2] In recent years, there has been a vigorous effort in defining the mathematical process and refining the solution techniques.[3] To determine the optimal trajectory of a continuously thrusting space vehicle, the differential equations of motion are usually nonlinear and, hence, require numerical techniques for obtaining a solution. The methods used are classified as either indirect methods (based on the conditions necessary to achieve mathematical optimality) or direct methods (which use only the differential equations governing the state and the specified boundary conditions). In both methods, successive changes in the initial solution are made in an attempt to satisfy the boundary conditions while extremizing a performance index. In this discussion, only indirect methods are considered.

Previously, only two factors were considered to affect the convergence characteristics of the indirect methods: the numerical integration procedure and the correction procedure that modified the unknown boundary conditions. Now, it is shown that the region in which the initial data are embedded can also have a significant effect upon the rate of convergence for an iteration process and the size of the convergence envelope.

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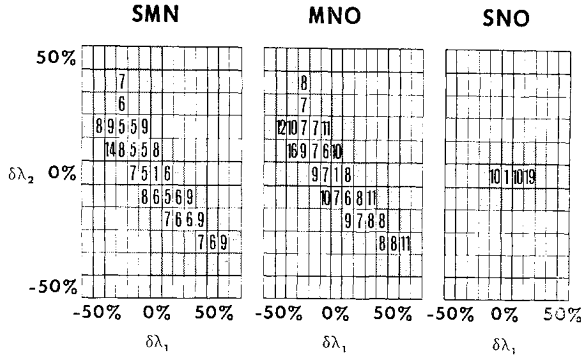

Fig. 1 Convergence envelope for MPF, final time optimal.

Computational Range

Much literature for solving problems on a computer is devoted to techniques for improving stability and reducing the effect of round off and truncation. The equations used are not restricted to a numerical reference frame, whereas the computer used to solve or express these equations is constrained to a fixed word length. The most suitable interval for performing arithmetic operations is the interval $[-1,1]$. This may be stated more formally as follows.

Theorem 1. The unit interval $[0,1]$ is more dense than any other unit positive interval for a fixed word length computer. That this is the case can be seen immediately if we consider that the computer has a mantissa represented by m bits and an exponent has e bits. Then the unit interval $[0,1]$ has 2^{m+e-1} different numbers or representations as an upper bound, whereas the interval $[a, a + 1]$ would have $2^{m-n} + 1$ numbers, where the representation for a requires n bits, $n > 0$. As a direct corollary, we have:

Corollary 2. The most dense biunit interval on a computer is the interval $[-1,1]$.

The direct implication that we would like to make is that the more bits we have to represent a number the more accurate the representation is. This leads to the following result:

Lemma 3. Let A and B be two computer words of length m and n, respectively, where $m > n$. Then a number can be represented more accurately in A than in B.

This lemma can be proved by considering the error between the two representations for an unending fraction. By choosing "nice" numbers the error can be the same, but for all other numbers the error is reduced by using the greater word length.

Normalization of the State Data

In solving problems on a computer, the units of measurement which are convenient in terms of physics or engineering may not be convenient for accuracy, as was shown in the previous section.

A convenient choice of normalized units for an Earth-Mars orbital transfer by Tapley and Lewallen[3] was the unit mass as the mass of the space vehicle, unit length as the Earth's orbital distance from the sun, and initial velocity as unity which established a time quantity and made the gravitational constant unity. This choice yields the fundamental quantities as mass, length, and velocity; all other units are derived from these.

Optimization Problem

In general the optimization problem can be stated as follows: determine the trajectory of the variables that are capable of controlling a dynamic system such that the functional

$$\hat{I} = \phi(x_0,x_f,t_f) + \int_{t_0}^{t_f} \theta(x,u,t)dt \qquad (1)$$